OPTIMAL DESIGN OF DISTRIBUTED SENSORS AND ACTUATORS MADE OF SMART MATERIALS FOR ACTIVE VIBRATION CONTROL *

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Abstract

We describe a design technique for optimal control in active structural vibration damping using smart materials. The vibration of a cantilever beam is stabilized by using distributed sensors and actuators. We model the beam by the Timoshenko beam model together with the distributed sensors and actuators. A control law using the weighted integration of vibration velocity is incorporated in the closed loop system. We propose a method to find the optimal layout design of the smart material so as to maximize the damping effect. An objective functional is defined based on the vibration energy of the system. The optimal shapes of the sensor and actuator are determined through minimizing the energy functional of the beam over the admissible shape function space subject to certain geometric constraints. An algorithm has been developed to determine the optimal sensor and actuator layout. This method can be generalized to the plate damping problem and more complicated structures as well.

Keywords: flexible structure control, distributed control, adaptive structures, sensors, actuators, smart materials.

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1 Introduction

An important issue in the control system design for flexible systems is the determination of the optimal number and location of the control system components: sensors and actuators as well as their backups. In general there is a larger number of candidate locations than available sensors and actuators. Based on experience and knowledge on structure dynamics and control objectives, a priori selection is usually available. However, this may not give the optimal effect on the closed loop system. Extensive experimental work is expected to justify the design. For discrete optimal sensor and actuator locations, a method based on the orthogonal projection of structural modes onto the intersection of the controllable and observable subspaces is introduced [3]. The controllability and observability Gramians are used to reflect the degrees of controllability and observability of an actuator/sensor pair. However, this method is based on a second order linear model. In [1], an objective function is defined based on the elements of the actuator influence matrix, and an optimization study is performed to compare the system performance. This work suggests that a relative even distribution of the actuators can lead to satisfactory results. Again, pointwise sensors and actuators are analyzed here.

The use of smart materials as sensors and actuators allows the adjustment of geometry and dynamical behavior of flexible space structures. It also provides means of signal processing by sensor's geometry. In [2], model sensors/actuators are proposed and developed. The sensors and actuators can provide signals related to certain elastic modes. It has been pointed out that the location of distributed sensors and actuators need to be placed away from the nodes of the specified elastic modes to be sensed or controlled to achieve maximum effect. The choice of the sensor and actuator shapes is also an important factor in system's performance. A question arises naturally: what are the optimal shapes of the distributed sensors and actuators made from smart materials?

We know that the flexible beam is an infinite dimensional system. In order to faithfully measure and control the system without using a truncated model, there is a need in designing control algorithms directly from the partial differential equation model to avoid spillover. We can develop certain performance measures to carry out the optimal design.

We consider the design issue associated with the vibration damping control of a cantilever beam. The beam is modeled as the Timoshenko beam. Both sides of the beam are covered by PVDF and PZT materials for sensing and actuation. Using the control algorithm developed in [4], the closed loop system can be asymptotically stabilized. Based on this result, we want to further determine the optimal layout of the continuous distributed sensors and actuators for the system based on minimizing the vibration energy of the system. We hope that this can lead to a general design methodology or at least provide a design guideline.

2 Problem formulation

We model the cantilever beam with the Timoshenko beam model which accounts for shear effects and rotary inertial. The Timoshenko model describes the physical behavior better than the Euler-Bernoulli model does especially for the high frequency vibration components. The actuator and sensor are the layers made of piezoelectric ceramic (PZT) and piezoelectric polymer polyvinylidene fluoride (PVDF) materials attached to both sides of the beam. Figure 1 shows the structure of the composite beam. The

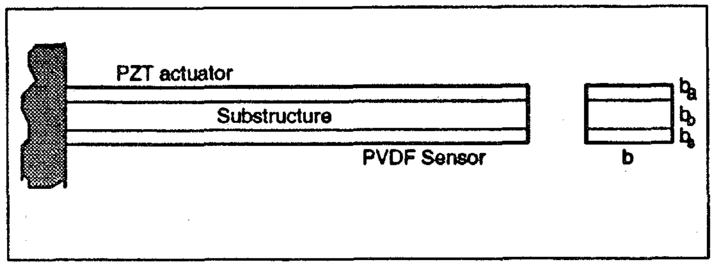

Figure 1: The composite beam

equations of motion are given [4] by,

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} = kAG(\frac{\partial^2 w(x,t)}{\partial x^2} - \frac{\partial \Phi(x,t)}{\partial x}), \tag{1}$$

$$\rho I \frac{\partial^2 \Phi(x,t)}{\partial t^2} = EI\frac{\partial^2 \Phi(x,t)}{\partial x^2} + kAG(\frac{\partial w(x,t)}{\partial x} - \Phi(x,t)) + c\frac{\partial V(x,t)}{\partial x}, \tag{2}$$

with boundary conditions

$$\begin{aligned} w(0,t) &= 0, \\ \Phi(0,t) &= 0, \\ \frac{\partial w(L,t)}{\partial x} - \Phi(L,t) &= 0, \\ EI\frac{\partial \Phi(L,t)}{\partial x} &= 0, \end{aligned} \tag{3}$$

where

$$EI = E_a I_a + E_b I_b + E_s I_s, \tag{4}$$

$$c = \frac{d_{31}}{h_a} K_a.$$

The constant c is determined by the PZT material property and the manufacturing process. The functions $w(x,t)$ and $\Phi(x,t)$ are the displacement of the centroid and orientation of the cross section of the beam. G is the Young's modulus in shear. EI is the bending rigidity with the subscripts b, a and s denoting beam, sensor and actuator layers respectively . A is the area of cross section. The distributed control $V(x,t)$ appears in (2) as the distributed bending moment.

Figure 2 shows the structure of the PVDF sensor. The sensor output is given by [4]

$$V_s(t) = K_s \int_0^L F(x) \frac{\partial^2 \Phi}{\partial t \partial x} dx \tag{5}$$

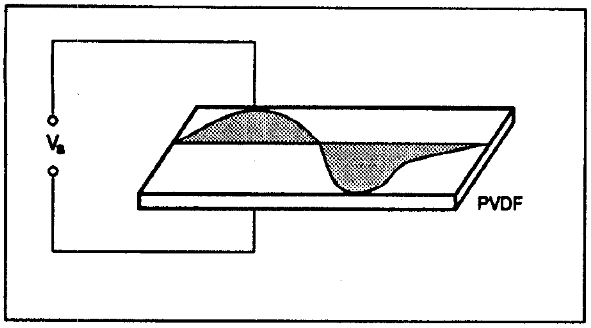

Figure 2: The PVDF sensor

which is a weighted integral of beam curvature along the longitudinal direction of the beam. The function $F(x)$, the sensor shape function, is the width of electrodes covering both sides of the PVDF sensor simultaneously. K_s is a constant determined by the sensor material. The feedback control is given by

$$V(x,t) = K_s v(x) \int_0^L F(x) \frac{\partial^2 \Phi}{\partial t \partial x} dx, \quad (6)$$

where $v(x)$ is the actuator shape function. Similarly to the sensor shape function, $v(x)$ is the width of the electrodes covering the surfaces of the PZT actuator.

We want to introduce active damping to extract vibration energy from the system. It is natural that the value of the vibration energy measures the amount of vibration. The energy functional is defined as

$$\begin{aligned} E(t) &= \frac{1}{2} \int_0^L \left\{ \rho A [\frac{\partial w}{\partial t}]^2 + \rho I [\frac{\partial \Phi}{\partial t}]^2 \right. \\ &\quad \left. + K[\frac{\partial w}{\partial x} - \Phi]^2 + EI[\frac{\partial \Phi}{\partial x}]^2 \right\} dx. (7) \end{aligned}$$

where

$$K = kAG.$$

The first two terms in the integral are kinetic energy of the beam due to the displacement and rotation. The third term is the energy due to shear. The last term is the stored energy from bending. The vibration energy defined above is a function of time t.

3 Optimal control and a numerical algorithm

Unlike point sensors and actuators, the geometry of the spatially distributed sensor and actuator has a function of preprocessing the sensor output signals and control weight. A judicious choice of the shapes can extract the desired signals and implement the control algorithm. We discussed in [4] a method of choosing the appropriate sensor and actuator shapes for active damping control by means of modal analysis. We would like to develop a systematic approach to deal with this problem here.

The control (6) is a functional of sensor and actuator shape functions and the weighted integral of the beam curvature. We have proved that the control (6) asymptotically stabilizes the system [4], i.e., we have

$$\lim_{t \to \infty} E(t) = 0. \quad (8)$$

We seek the optimal control in the sense that the energy functional is minimized over all the possible sensor and actuator shapes. Our task here is to find the optimal sensor and actuator shape functions $v(x)$ and $F(x)$ so as to minimize the energy functional (7). The problem is to find functions $v_0(x)$ and F_0 such that

$$\begin{aligned} J[T, v_0, F_0] &= \min_{v \in \mathcal{V}, F \in \mathcal{F}} \frac{1}{2} \int_0^L \left\{ \rho A [\frac{\partial w}{\partial t}]^2 \right. \\ &\quad + \rho I [\frac{\partial \Phi}{\partial t}]^2 + K[\frac{\partial w}{\partial x} - \Phi]^2 \\ &\quad \left. + EI[\frac{\partial \Phi}{\partial x}]^2 \right\} dx, \quad (9) \end{aligned}$$

where $\mathcal{V}$ and $\mathcal{F}$ are the sets of all the admissible actuator and sensor shape functions. The admissible functions here depend on geometry of the structure. For beam and plate like structures, the geometry is usually simple. Since the region of the beam which could be covered with smart materials is assumed to have length L and width b, the sets $\mathcal{V}$ and $\mathcal{F}$ contain the collection of all the piecewise continuous curves within this region. The optimization hence has a geometric constraint. The functions $v(x)$ and $F(x)$ denote the width of the electrodes covering the smart materials; we have $0 \leq v(x) \leq b$ and

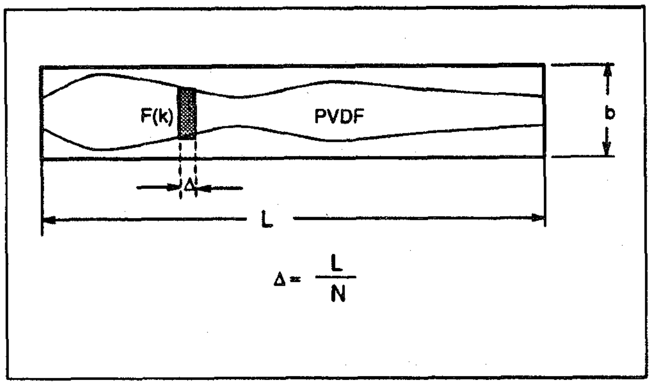

Figure 3: Discretizing the sensor and actuator layout

$0 \leq F(x) \leq b$. We are interested in observing the amount of energy at time T.

For numerical solution of a shape optimization problem, one typically starts by guessing an initial design. One then discretizes the elastic problem using finite elements or using difference method or a Galerkin procedure. After discretization, the optimal design problem becomes a large nonlinear programming problem. Different routines are available for working on the later.

We start our numerical scheme by discretizing the region along the longitudinal spatial variable x as in Figure 3. Let N be the total number of segments with equal size, then the width of each segment is L/N. The discretized shape functions $v(x)$ and $F(x)$ assume constant values $v(k)$ and $F(k)$ inside the k^{th} element. We thus have piecewise constant functions $v(k)$ and $F(k)$ with $k = 1, 2, ...N$. The $2k$ members of $v(x)$ and $F(k)$ become the optimization parameters. We then compute the distributed control $V(k,t)$, $k = 1, 2, ...N$, based on the initial conditions of the system and the discretized shape functions. The time response to the input can be computed through solving the equations of motion numerically. This procedure yields the value of the cost functional J at time T. An optimal routine shall be followed to search and adjust the piecewise constant sensor and actuator shape functions toward reducing the value of the cost functional (9). The new shape functions are then used to generate the system input $V(k,t)$ again. This procedure is repeated until the optimum criterion is met. The resulting $v(k)$ and $F(k)$ can be smoothed to give the final optimal shape functions $v_0(x)$ and $F_0(x)$ for the sensors and actuators. The algorithm is given in Figure 4.

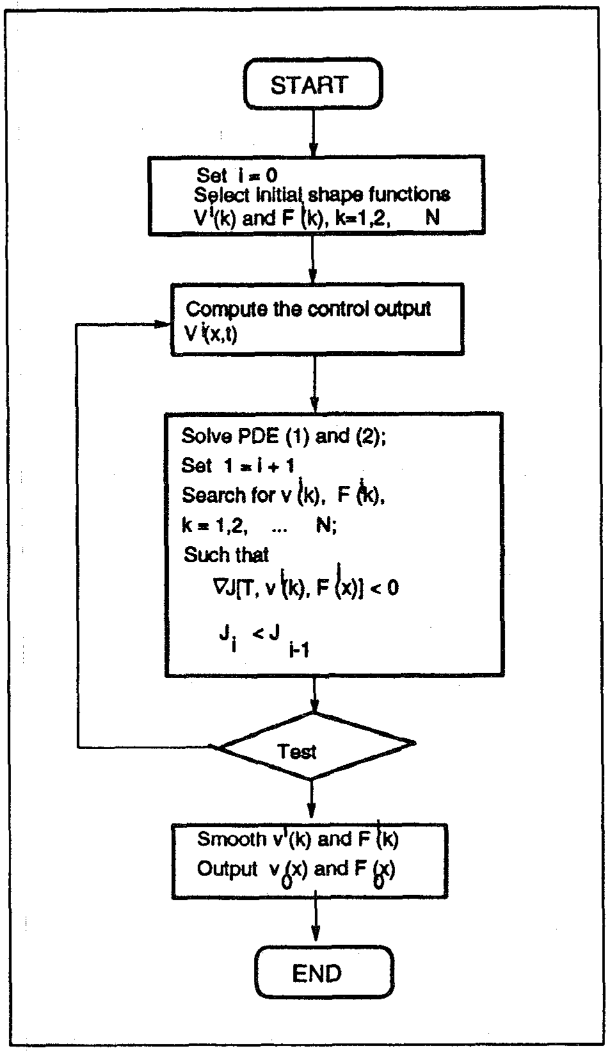

Figure 4: Optimization algorithm

4 Other considerations

The optimization scheme can be used to deal with more complicated structures. The sensors and actuators may be piecewise continuously distributed on the structures. When structures contain both distributed and pointwise sensors and actuators, our formulation still holds. The corresponding shape functions $v(x)$ and $F(x)$ become both piecewise continuous and pointwise in the relevant regions.

In terms of computation, the bottleneck is the simulation of the systems governed by partial differential equations. Different methods can be implemented to solve the partial differential equations (1) and (2).

Modal sensors and actuators can be designed through optimization as well. This may reduce the influence of leak-through, i.e. the crossover effect among different modes, to improve the overall performance. Different performance measures and cost functions are required to formulate the optimization problems.

5 Conclusions

We have developed a method to facilitate the optimal design of active vibration damping using smart materials. The optimal algorithm described above can be expected to yield reasonable good design for the layout of the distributed sensors and actuators. Although the algorithm is developed based on the beam model, the method can be extended to the plane case. This procedure is expected to work for the cases with irregular geometry or nonuniform structural material as well.

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