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Short Papers

Estimation of Traffic Platoon Structure from Headway Statistics

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***Abstract*—This paper deals first with the modeling of urban traffic headway statistics. It is shown that a composite distribution based on the convex combination of a lognormal and a shifted exponential distribution gives a good fit to observed traffic data. This statistical model is then used to generate a model for the formation and passage of "platoons" of vehicles. It is shown that the problem of estimating the time at which a "platoon" passes a detector, as well as the number of vehicles in the "platoon," corresponds to the point process disorder problem. An optimal estimator for the platoon size and passage time, based on detector data, is then derived via known results for the point process disorder problem. It is shown that the computations required by this estimator can be performed in a microprocessor. Furthermore, the estimator is tested against the UTCS-1 traffic simulator and performs very well. Parameter sensitivity analysis of the estimator is presented. Finally, the use of these results to improve the filter/predictor described in a companion paper, and vice versa, is explained.**

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I. INTRODUCTION

In a companion paper [14] we noted that there is considerable current interest in the development of computer-based systems for the control of urban traffic. In addition, we explained that these systems generally do not make much use of data acquired in real time because of difficulties in estimating relevant traffic parameters from such data. Finally, we presented three procedures for estimating queue length at a signal from detector data.

This paper presents a procedure for estimating the time at which a "platoon" of traffic passes a detector as well as the number of vehicles in the "platoon." Roughly, a "platoon" is a group of vehicles that move with similar velocities and comparatively small spacing. Although platoons of vehicles are observed in freeway traffic as well, this phenomenon is a rather fundamental characteristic of traffic in an urban network and is greatly influenced by the traffic signals. Indeed the periodic variation of traffic lights tends to group vehicles into platoons. Traffic engineers have long exploited this behavior by using the maximum through-band synchronization scheme. The technique consists of offsetting the green phase of successive traffic lights, with respect to each other, to regulate groups of moving vehicles at some desired speed without stopping. Thus, it is believed that estimates of platoon size and passage time may be an especially relevant traffic parameter for control purposes.

Furthermore, it was explained in the companion paper that it is very desirable to have adaptive queue estimators. Such adaptive estimators need information about estimation errors that is largely independent of the estimator itself. To clarify this point and for future references we consider in Fig. 1 two successive signalized traffic intersections. Loop

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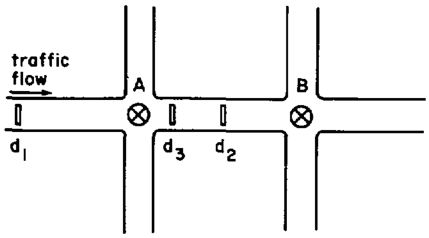

Fig. 1. Typical succession of traffic lights and detectors in urban traffic.

detectors d_i are typically located so that d_1,d_2 are relatively close to the downstream light (15–20 vehicle lengths) and serve as the observations for the queue estimators [5]; detector d_3 can be located either directly after traffic light A to provide observations on vechicle discharge (departures) from traffic light A, or near the middle of the link AB to provide observations about the structure of upcoming traffic flow towards intersection B. In the former case, the platoon estimators described here can provide an independent, delayed estimate of the queue at traffic light A. Thus, the platoon estimator also has potential utility as a device for making the queue estimator adaptive (actually a crude estimator of this sort is currently used in UTCS-1 for exactly the same reason). In the latter case observations from detector d_3 are very important (as well as those from detector d_2) to the traffic controller B. Indeed a platoon estimator in this case can provide advanced information to controller B about the structure of the upcoming traffic demand (e.g., platoon size, gaps, etc.) resulting in more efficient control.

In Section II we present the models for platoon formation and flow. It is shown that vehicle headway statistics form the basis for the development. Furthermore, it is shown that two different interpretations of the same simple stochastic model lead to a model for urban traffic on one hand and to a model for freeway traffic on the other. In Section III we show that estimation of platoon passage time corresponds to the point process "disorder" problem. The solution to this problem is then given and means whereby the required calculations can be performed in a microprocessor are explained. The test and evaluation of the resulting estimator using the UTCS-1 simulation are briefly discussed in Section IV. In Section V results pertaining to the parameter sensitivity of the proposed estimator are presented. Finally, in Section VI a brief description of our current research on alternative solutions to this estimation problem and on adaptive urban traffic control is given. For more detailed descriptions of the results presented here we refer the reader to [5], [19].

II. Models for Headway Statistics and Resulting Traffic Models

It has been recognized that one of the most important components in the description of traffic flow is the distribution of headways. Although several definitions of headway exist, we will always mean the time difference between the passage of the leading edges of successive vehicles. The statistical distribution of headways has been studied extensively since the early days of traffic control. It is natural for our work for two reasons.

1) It is relatively easy to collect headway data from the existing detectors.

2) The statistical desription of headways (interarrival times in the point process jargon) is the essential part in modeling the underlying point process and is the point of departure for the modern theory of estimation for point processes [1]–[4].

For a complete description of the traffic process we need to include the speed measurements provided by the detectors [5]. This is the mark (in point process jargon [13]) of the point process that characterizes traffic detector output. Such measurements and a model similar to ours have been effectively utilized in [20], [21] to describe freeway traffic. It is worth emphasizing that [20] provides a substantial validation of the aforementioned model. In this paper, due to simplifying considerations, we consider only headway statistics. Speed statistics and a more complete development based on a mixed headway-speed model will be given elsewhere.

Most of the prior work on headway statistics was concerned only with the probability density for headways. We include here a very brief survey of this work. Our reports in [5] and [19] contain a considerably more detailed survey.

In one of the earlier studies Adams [6] proposed a negative exponential headway probability density. The model broke down when traffic was no longer freely flowing (e.g., due to traffic lights or difficulty in passing). One of the shortcomings of the negative exponential density occurs at very short headways. This can be rather easily corrected with a displaced negative exponential density. A more fundamental limitation of this density, however, is its failure to describe the smaller variability in the headways observed in groups of vehicles that follow each other (i.e., platoons). As a result, although the displaced negative exponential density is universally accepted [6]–[12], [20], [21] as a very good model for relatively long headways (i.e., corresponding to freely flowing and nonfollowing vehicles) different types of densities were proposed for short headways. Such models include Erlang, gamma, and lognormal densities. From these so-called single density models the lognormal density

$$p(h)=\begin{cases} \dfrac{1}{\sigma h\sqrt{2\pi}}\exp\left(-\dfrac{(\ln h-u)^2}{2\sigma^2}\right), & h\geq 0 \\ 0, \quad h<0 \end{cases} \tag{2.1}$$

(where μ,σ^2 are the mean and variance of $\ln h$), or shifted lognormal density gave the best results in fitting observed data from platooning vehicles [8], [9]. There are various justifications for these findings about the lognormal density. The primary reason is that multiplicative, independent, identically distributed errors by various drivers attempting to follow each other combine to give a lognormal density.

The implicit concern about the different statistical behavior of short and long headways eventually leads to the so-called composite density models which give better fits to observed data than single density models [11], [20], [21]. This type of model assumes a structure of traffic consisting of two subpopulations: one corresponding to following traffic (i.e., traffic grouped in platoons) and one corresponding to nonfollowing traffic (i.e., freely flowing vehicles or leaders of platoons). The headway probability density assumes then the form

$$p(h)=\psi p_f(h)+(1-\psi)p_{nf}(h) \tag{2.2}$$

where

p_f = following headway probability density function (short headways).
p_{nf} = nonfollowing headway probability density function (longer headways). It is usually a displaced negative exponential density.
ψ = degree of interaction.

Since headway is dependent on traffic flow, the degree of interaction incorporates this dependency. For light traffic, for example, ψ equals zero yielding a composite density that is a displaced negative exponential. There are several interpretations one can give to ψ and we shall return to this point later. It has been found [10], [20] that p_f does not depend on the position of the vehicle within the platoon or on the size of the platoon.

Such a composite type model has been recently described by Branston [11]. This model provided excellent fit to data from various traffic flow situations [11]. It utilizes a lognormal density (2.1) for following headways and the random platoon assumption of Miller [7] (that is, the gaps between platoons follow an exponential density). The resulting probability density for headways has the form

$$p(h)=\begin{cases} \psi g(h)+(1-\psi)\lambda\exp[-\lambda h]\int_0^h g(x)\exp(\lambda x)\,dx, & h\geq 0 \\ 0, \quad h<0 \end{cases} \tag{2.3}$$

where g is the lognormal density (2.1).

There are several reasons that make this model attractive: 1) the parameters introduced by the model are natural and are important parameters for filter/prediction and (or) control, 2) the model can accommodate all traffic conditions (light, moderate, heavy) and is valid for practically all ranges of traffic flow and speed (a property that has been verified from real data and which is not true for simple models), 3) the distributions involved imply underlying stochastic processes that can be completely described by a finite number of moments (at most two), an important fact for the development of simple but effective filter/predictors, and 4) the two basic assumptions of the model are the lognormal following headway distribution and the exponential interplatoon gap which as we discussed earlier are very well documented and validated.

A computational drawback of this model is the rather complicated expression for the nonfollowing headway density (2.3). Our results in [5], [19] indicate that an equally valid model is obtained if a displaced negative exponential is used to model nonfollowing headways. This is further supported by the wide acceptance of this density as an appropriate model for longer headways. As a result of these considerations the model adopted for the first-order headway density is given by (2.2) where p_f is as in (2.1) and p_{nf} has the form

$$p_{nf}(h)=\begin{cases}\lambda e^{-(h-\tau)\lambda}, & h\geq\tau\\ 0, & h<\tau.\end{cases} \tag{2.4}$$

We would like to emphasize that Breiman *et al.* in [20], [21] arrived at a similar model for freeway traffic. Their model, although not explicitly specifying the following headway density as lognormal, was carefully validated with a sizable data base. As discussed earlier there are very good reasons for our proposal of the lognormal density and furthermore, our results can be easily modified to accommodate other densities. It is then apparent that our proposed model applies equally well to urban and freeway traffic.

The model requires five parameters for the headway density: ψ,λ,τ,μ, and σ. To completely specify the model for a particular link or section of a link in a traffic network, it is important to understand the variation of these parameters with respect to traffic flow and speed. Both Branston [11] and Breiman *et al.* [20], [21] report that μ,σ are fairly insensitive to traffic flow level while varying from lane to lane and link to link. The parameters ψ and λ depend on traffic flow and are rather easily estimated [11], [20], [21] if one utilizes velocity (speed) statistics as well. Finally, τ varies between 0.25 and 4.00 s and can be easily estimated [20].

The probability density given does not provide in general a complete description of the headway stochastic process at a particular point in a traffic network. Higher order probability density functions are also needed because there may exist a correlation between successive headways. On the other hand, we know from point process theory that interarrival time statistics completely characterize the process and, in particular, can be used to determine the "rate" of the process [4], [13]. This rate plays a central role in estimation. In [5], [19] we have developed a model that utilizes correlated following headways as observed by Buckley [12]. To simplify computations and based on evidence provided in [20] we analyze for the balance of this paper a model which employs uncorrelated following headways. We are currently investigating the effects of this approximation which appear to be insignificant. Since nonfollowing headways are clearly independent, the resulting model assumes independent successive headways. So we have a self-exciting process with memory 1 [13]. As a result of these simplifications the headway process is characterized by the first-order density (2.2).

We developed two interpretations for the mixed headway model. The first model is intended for use in estimating gross traffic patterns for the slow updating of traffic flow parameters (both in urban and in particular freeway traffic). In such a case ψ, which should be interpreted as the probability that a particular headway is a following headway, should be constant for long time intervals. The second model is intended for use in urban nets with small average link lengths and traffic signals. In such cases it is crucial to model the periodic formulation and propagation of platoons or queues as modulated by traffic lights. Then ψ is modeled as a time function with values 1, corresponding to passing of a platoon or a queue discharge, and 0 corresponding to nonfollowing freely flowing traffic.

We call the first model the *average mixed headway model*. The point process it characterizes has rate

$$\lambda_t(N_t=n,T_n)=\frac{p(t-T_n)}{1-\int_0^{t-T_n}p(x)\,dx}\qquad\text{for } t\geq T_n \tag{2.5}$$

where p is given by (2.2), N_t = the accumulated detector counts up to time t, T_n = the time of the nth detector activation, and λ_t = the rate of the detector point process at time t (see Section III).

$$h(t)=\frac{p(t)}{1-\int_0^t p(x)\,dx} \tag{2.6}$$

is sometimes referred to as the hazard function in birth or renewal process jargon. Our results indicate that filters/predictors behave well if the hazard function is chosen appropriately. This suggests the alternative: derive filter/predictors by appropriate choice of the hazard function and make them adaptive by tuning the hazard function to the traffic flow pattern.

We call the second model the *switching rate mixed headway model*. This model is based on the switching of ψ between 0 and 1. As a result the point process will have two rates. The following headway rate is

$$\lambda_f(N_t=n,T_n)=\frac{g(t-T_n)}{1-\operatorname{erf}\left[\dfrac{\ln(t-T_n)-\mu}{\sigma}\right]}\qquad\text{for } t\geq T_n \tag{2.7}$$

where g is the lognormal density (2.1). For the nonfollowing headway process the rate is given by [using (2.4)]

$$\lambda_{nf}(N_t=n,T_n)=\begin{cases}\lambda & \text{if } t-T_n\geq\tau\\ 0 & \text{if } t-T_n<\tau.\end{cases} \tag{2.8}$$

Typical hazard functions for the following and nonfollowing headway process are shown in Fig. 2. Some of these computations are used later in the disorder problem for point processes. These computations complete the description of the headway process model.

A model can now be developed for urban traffic flows based on the headway model adopted. Each link is divided in sections in accordance with the detectorization of the link. For each section of the link the input and output traffic flows will have headway distributions as described above. Notice that the headway distribution model can vary (and it should) from lane to lane [20], [21]. The required parameters of the model will be estimated at appropriate intervals from actual data, or from historical data as required. The effect of the link on the flow of traffic will be modeled by altering the parameter values of the model as we move from section to section.

III. Platoon Structure Estimation

The filter/predictors developed in this section are based on some fundamental recent results in point processes as developed by Boel, Varaiya, and Wong [1], [2], Segall, Davis, and Kailath [3], and Davis [4]. The approach we have taken in Section II is motivated by the work of Davis who demonstrated in [4] that a complete statistical description of interarrival times is adequate for filtering/prediction problems based on point process observations. This has both a theoretical appeal and is significant for practical applications where interarrival time statistics (i.e., headway statistics in the traffic context) are rather readily available from experiments.

A setting for continuous-time filtering based on point process observations is as follows. The signal process is modeled by the stochastic differential equation

$$dx_t=f_t\,dt+dv_t \tag{3.1}$$
$$x(0)=x_0$$

where v_t is a martingale with respect to the σ-algebra $\mathfrak{B}_t$ which is generated by the past sample paths (i.e., $s\leq t$) of the signal and point observation processes (the analog of $\mathfrak{B}_{t-1}$ in our companion paper [14]). Usually f_t is a function of the past of the signal and point observation processes. Furthermore, the observation point process is modeled by

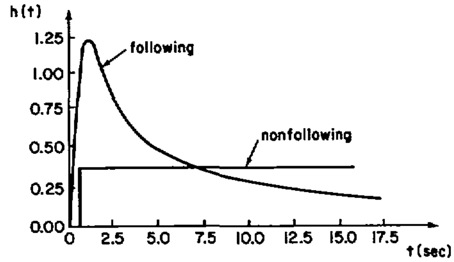

Fig. 2. Typical hazard function for following and nonfollowing headways.

$$dN_t = \lambda_t\,dt + dw_t \tag{3.2}$$

where w_t is also a martingale with respect to $\mathfrak{B}_t$ and λ_t is the "rate" of the process. Usually λ_t is a function of the past of the signal and point observation process. Let $\mathfrak{F}_t$ be the σ-algebra generated by the past of the point observation process (i.e., $N_s, s \leq t$). Then the minimum error variance estimate of the signal x_t given the past of the point observation process is

$$\hat{x}_t \triangleq E\{x_t|\mathfrak{F}_t\} \tag{3.3}$$

and is given by

$$d\hat{x}_t = \hat{f}_t\,dt + (\hat{\lambda}_t)^{-1}E\left\{x_t(\lambda_t - \hat{\lambda}_t) + \frac{d}{dt}\langle v, w\rangle_t | \mathfrak{F}_t\right\}\cdot(dN_t - \hat{\lambda}_t\,dt) \tag{3.4}$$
$$\hat{x}_0 = E\{x(0)\}$$

where "^" denotes conditional expectation with respect to $\mathfrak{F}_t$, and

$$dv_t = dN_t - \hat{\lambda}_t\,dt \tag{3.5}$$

is the so-called innovation process of the observation process. This is a general result and for a particular problem the various terms have to be computed and substituted in (3.4), which is not recursive in general.

Although several filtering/prediction problems of relevance to urban traffic control problems can be formulated in the above framework, we concentrate on the estimation of traffic patterns (i.e., passage time of platoon or queue). From Section II the point process observed by a traffic detector is a mixture of two point processes each with a different rate process; one associated with following vehicles (i.e., in platoons or queues) (2.7) and a different one associated with nonfollowing vehicles (2.8). The rate of the overall process switches between these two rates (switching rate mixed headway model). Estimates of the switching times can be very useful for the following reasons (see Section I): 1) they determine the traffic flow pattern, and if transmitted to downstream detectors and traffic light controllers, will lead to improvement in filtering/prediction and control of subsequent links; and 2) a common problem with queue estimators is the errors from traffic cycle to traffic cycle due to vehicles trapped by the red light or vehicles passing during the amber to red transition. By effectively estimating from the first downstream detector (i.e., the one immediately after the traffic light in Fig. 1) the time when the last queuing vehicle has passed that detector a reinitialization of the upstream queue estimator can be implemented to correct, cycle to cycle, propagation of cumulative errors.

In a different, traffic oriented problem, we are often interested in estimating or detecting the times when large changes in the rate process occur. This is often related to an incident in a freeway (or urban traffic link). This is the incident detection problem and will be treated elsewhere.

All the above problems can be formulated in the context of the so-called point process "disorder" problem. Namely, we observe a point process N_t which is governed by a rate process λ_t^0 until some random time T (called the "disorder" time), and by a different rate λ_t^1 after this time. The problem is then to estimate the switching time T from the observations of N_t only. This problem has been studied by Shiryaev [15], Galchuk and Rozovsky [16], Davis [17], and in complete generality by Wan and Davis [18]. We follow the last two references in the development presented here.

We first need to establish the structure of the problem as in (3.1), (3.2). Let us define

$$x_t = I_{\{t > T\}} \tag{3.6}$$

where $I_{\{t>T\}}$ is the characteristic function of the set $\{t > T\}$. So x_t indicates by switching from 0 to 1 the "disorder" time. Of the several cases considered in the literature, the appropriate one for the traffic problems discussed earlier is the following: the switching time T coincides with one of the detector activation times, T_i (occurrence times). In general, and in particular for traffic problems, the events $\{T = T_i\}$ may not be independent from the underlying point process N_t. Let

$$p_i = \Pr\{T = T_i\}; \qquad q_i = \frac{p_i}{\sum_{k > i} p_k} \tag{3.7}$$

$$q_t = \sum_i q_i I_{\{T_{i-1} < t < T_i\}}. \tag{3.8}$$

By some calculations which can be found in [18] or [19], one can then show that

$$dx_t = (1 - x_t)q_t\lambda_t^0 + dv_t \tag{3.9}$$

and

$$dN_t = ((1 - x_t)\lambda_t^0 + x_t\lambda_t^1)\,dt + dw_t \tag{3.10}$$

which are of the same form as (3.1) and (3.2). The filter (3.4) now becomes

$$d\hat{x}_t = -(\lambda_t^1 - \lambda_t^0)\hat{x}_t(1 - \hat{x}_t)\,dt + \frac{(\lambda_t^1 - \lambda_t^0)\hat{x}_t(1 - \hat{x}_t) + q_t\lambda_t^0(1 - \hat{x}_t)}{(\lambda_t^1 - \lambda_t^0)\hat{x}_t + \lambda_t^0}\,dN_t$$
$$\hat{x}_0 = E\{x_0\} = p_0. \tag{3.11}$$

Note that

$$\hat{x}_t = E\{x_t|\mathfrak{F}_t\} = E\{I_{\{t>T\}}|\mathfrak{F}_t\} = \Pr\{T \leq t|\mathfrak{F}_t\} \tag{3.12}$$

so that (3.11) computes the probability that the switch has occurred prior to time t given the detector data up to time t. When there is dependence between the events $\{T = T_i\}$ and $\{N_t\}$ some simple arguments [18], [19] lead to (3.11) with the exception that

$$q_t = \sum_i q_i(t, T_1, T_2, \cdots, T_{i-1})I_{\{T_{i-1} < t < T_i\}}. \tag{3.13}$$

Thus, the only change needed to accommodate dependence between $\{N_t\}$ and $\{T = T_i\}$ is to let q_i be a function of t and the prior T_i. Given explicit expressions for the two rates λ_t^0, λ_t^1, then (3.11) is an implementable, nonlinear filter. Using expressions (2.7), (2.8) we proceed to derive explicit equations for the filter. Between detector activations ($dN_t = 0$)

$$\frac{d\hat{x}_t}{dt} = -(\lambda_{nf} - \lambda_f)\hat{x}_t(1 - \hat{x}_t), \qquad T_{i-1} < t < T_i. \tag{3.14}$$

This equation can be solved explicity [5], [19] to give

$$\hat{x}_t = \left[1 + \frac{1 - \hat{x}_{T_{i-1}}}{\hat{x}_{T_{i-1}}}\exp[\lambda(t - T_{i-1} - \tau)u(t - T_{i-1} - \tau)] \cdot \left(1 - \operatorname{erf}\left[\frac{\ln(t - T_{i-1}) - \mu}{\sigma}\right]\right)\right]^{-1} \quad \text{for } T_{i-1} < t < T_i \tag{3.15}$$

where u is the unit step function. On the other hand, when $t = T_i$ (i.e., at detector activation times) the estimate has a jump discontinuity with size equal to the coefficients of dN_t in (3.11)

$$\hat{x}_t - \hat{x}_{t-} = (1 - \hat{x}_{t-})\frac{(\lambda_{nf}(t-) - \lambda_f(t-))\hat{x}_{t-} + q_{t-}\lambda_f(t-)}{(\lambda_{nf}(t-) - \lambda_f(t-))\hat{x}_{t-} + \lambda_f(t-)}. \tag{3.16}$$

Thus, the filter is actually implemented as follows: 1) between detector activation times (3.15) is used, 2) at detector activation times the jump discontinuity is computed from (3.16), and 3) the error function appearing in (3.15) is computed by a five term series expansion.

Finally, for the implementation of the filter we need to determine the deterministic function q_t, which in our case is given by (3.8) and therefore we need to specify the p_i's in (3.7). It is clear from the definition of the p_i's that the information carried by them is identical to the probability density for queue length. That is, the output of queue estimators that compute the probability density for queue length (see our companion paper [14]) can be used to compute the values for p_i. For simplicity and to obtain a "worst case" type evaluation of the filter performance, we used a uniform probability density over the maximum possible queue length. That is,

$$p_i = \frac{1}{N} \qquad i = 1, \cdots, N \tag{3.17}$$

where N was the maximum queue length allowed (i.e., the distance in car lengths of the upstream detector from the traffic light). This ignores any information that we may have about the queue from other detectors and forces the filter to use only the information available from the detector data. On the other hand, if we had an estimate of the queue size, say 8, we could define the p_i's to be centered around 8, i.e.,

$$p_i = \begin{cases} 0 & i = 0, 1, 2, \cdots, 6 \\ 0.0125 & i = 7, 9 \\ 0.975 & i = 8. \end{cases} \tag{3.18}$$

The result of this more selective p_i will be a bias for the estimate around 8 if no disturbance (such as queue dispersion or platoons joining the queue) occurs. On the other hand, if a disturbance occurs the output of the filter will show it.

IV. Platoon Estimator Evaluation

The estimator depends on four parameters. The first parameter is λ, the mean arrival rate for free-flowing traffic. The second parameter is τ, the displacement of the negative exponential density. The third and fourth parameters, μ and σ, define the lognormal distribution associated with following headways.

In all of the tests, the parameters were held at $\lambda = 0.10$, $\tau = 0.0$ s, $\mu = 1.0$, and $\sigma^2 = 0.1681$. The value of σ was chosen to match Branston's value [11] which was obtained for freeway traffic. He showed that σ did not vary very much over different traffic flow levels. The value of μ was chosen so that the mean headway between successive vehicles in a platoon, as given by the lognormal distribution, would be 2.9 s. The value of λ was chosen so that the mean headway for nonfollowing vehicles would be 10 s.

The estimator, which we will denote by *PE*, must be given an initial estimate of the probability of each feasible number of vehicles in the platoon. In other words, the initial condition of the estimator is also an input. This initial condition is interpreted as the *a priori* probability distribution of the number of vehicles in the platoon. In the actual operation of *PE*, this *a priori* probability distribution can come from any one of several sources. For example, if *PE* is being used to refine the estimate of the queue, then *PE* can be initialized with the conditional probability of each queue length obtained from one of the estimators given in our companion paper [14]. A good choice of initial condition will greatly enhance the accuracy of *PE*.

In all of our tests *PE* was initialized with a uniform probability for any number of vehicles in the platoon up to fifteen. The uniform distribution was chosen because it provides essentially no *a priori* information. Thus, the performance of *PE* in these tests depends only on the data from the detector and is not biased by either accurate or erroneous foreknowledge. In a real application the performance would almost certainly be better.

The estimator actually computes, in real time, the probability that the platoon has passed, given the data up to time t. This is plotted for the first and second signal cycles on link 6–7 in test 1 in Figs. 3 and 4, respectively. These data were obtained from FHWA's UTCS-1 simulation. A more detailed description of the simulation and of test 1 can be found in our companion paper [14], our report [5] and [19]. For our purpose here, it is sufficient to note that the detector is 290 ft downstream from the traffic light that causes the platoon to form.

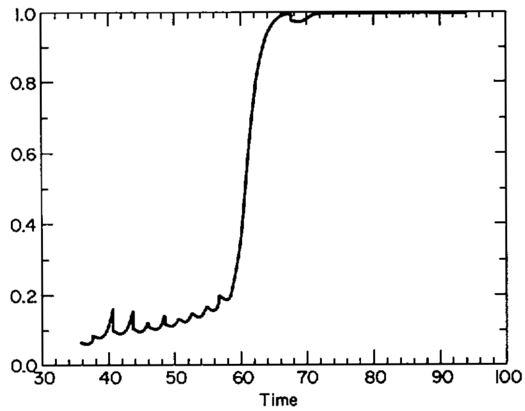

Fig. 3. Estimate of the probability that platoon has passed. Data are from test 1, link 6–7, midblock detector, cycle 1.

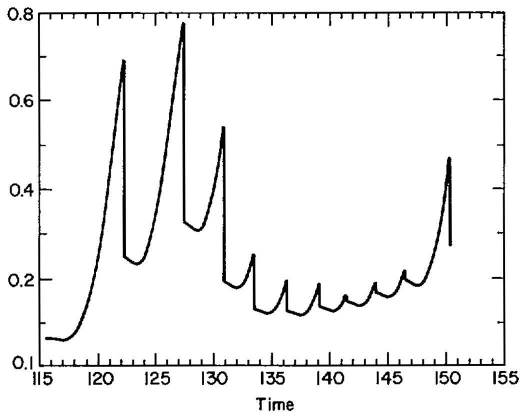

Fig. 4. Estimate of the conditional probability that platoon has passed. Data are from test 1, link 6–7, midblock detector, cycle 2.

In order to evaluate *PE* conveniently, the detailed data from the figures are reduced to a scalar estimate in Table I. Two estimates are obtained.

1) The estimate is the number of vehicles that have passed the detector at the first instant that the estimated probability that the platoon has passed the detector exceeds 0.7. This is called the *threshold estimate*.

2) The estimate is the number of vehicles that have passed the detector at the time of the largest increase in the estimated probability that the platoon has passed the detector. This is called the *maximum jump estimate*.

Of course, reducing the output provided by *PE* into a single, scalar estimate throws away a great deal of information.

Note the apparent poor performance of the estimator in Cycles 1, 4, and 5. In fact, the errors are due to a platoon from upstream joining the end of the platoon formed by the traffic signal and then the combined platoon crossing the detector. This is quite apparent in Fig. 3 where the largest headway among the first ten vehicles to cross the detector is 3.2 s. Thus, the estimator would be exactly correct if it estimated a platoon of 10 vehicles. Thus, the error is only one vehicle. In Cycle 4, the largest headway among the first nine vehicles is 4.2 s. And, in Cycle 5, the largest headway among the first eleven vehicles is 2.9 s. Thus, there is really only one vehicle error in the estimate of platoon size on Cycle 4 and no error in Cycle 5. The anomaly in the table is caused by the fact that "actual queue" means the number of stopped vehicles while, in fact, vehicles can join the platoon without ever coming to a complete stop.

Table II summarizes the results of a much more favorable traffic situation. The upstream traffic signal is 800 ft away so that there is a

TABLE I
PERFORMANCE OF PLATOON ESTIMATOR[a]

Cycle Number	Threshold Estimate	Maximum Jump Estimate	Actual Queue
1	11	11	2
2	3	2	2
3	3	3	3
4	10	10	1
5	0	11	1
6	3	3	2

[a]The data are from test 1, link 6–7. The threshold used was 0.70.

TABLE II
PERFORMANCE OF PLATOON ESTIMATOR[a]

Cycle Number	Threshold Estimate	Maximum Jump Estimate	Actual Queue
1	6	7	6
2	6	8	8
3	7	7	7
4	7	7	7
5	9	9	9
6	1	6	7

[a]The data are from test 5, link 6B–5B, lane 2, the stop line detector. The threshold used was 0.70.

relatively large gap between successive platoons. Furthermore, the detector is located near the downstream stop line so the flow over the detector is in clearly defined platoons.

These results indicate that *PE* is an accurate estimator of the number of vehicles in a platoon. Furthermore, it accurately determines whether or not the queue emptied on a cycle by cycle basis.

V. PARAMETER SENSITIVITY AND ESTIMATION

To complete the analysis of the estimators presented we need to determine methods which compute adaptively the filter parameters, and we also need to evaluate the response sensitivities to these parameters. These are rather hard analytical problems and some partial results have been obtained in [19], where we refer for further details. We present here our most significant findings in summary.

The filter parameters are λ, τ, μ, and σ. In Section I we discussed briefly their variability with traffic flow and location [11], [20], [21]. Our initial approach to the automatic identification of these parameters centered around outlier tests [22]. To estimate μ and σ, for example, we used the fact that if we take the natural logarithm of headway data the following headways will be distributed according to a Gaussian density while the nonfollowing ones will constitute a contamination with relatively higher values. The standard outlier estimation technique involves the following steps.

1) Let $k=1$, and compute $\bar{x}_1$ and $\bar{s}_1^2$ for the entire sample of headways (after taking natural logarithms).

2) By a two-sided outlier test, discard those observations outside the $2\bar{s}_k^2$ neighborhood of $\bar{x}_k$.

3) Let $k=k+1$ and compute $\bar{x}_k$ and $\bar{s}_k^2$ of the remaining sample.

4) If $|\bar{x}_k-\bar{x}_{k-1}|<\epsilon_1$ and $|\bar{s}_k^2-\bar{s}_{k-1}^2|<\epsilon_2$, then set $\mu=\bar{x}_k$ and $\sigma^2=\bar{s}_k^2$. Otherwise go to step 2).

It is not difficult to see that the algorithm converges. The resulting values were satisfactory only for samples including more than 50 percent following headways [19]. The estimate of μ was always more accurate than that of σ. These findings indicate that if one first estimates λ and τ and then applies the outlier method to the low end headway data, μ and σ would be easily estimated. Fortunately, this can be done by first fitting a displaced negative exponential to the tail of the observed headway density. This was done, for example in [20], employing the Kolmogorov–Smirnov test in an iterative way. The results were very satisfactory. It appears, therefore, that the combination of these two techniques produces reliable estimates of λ, τ, μ, and σ. However, the development of on-line or real-time parameter estimation techniques remains an open problem.

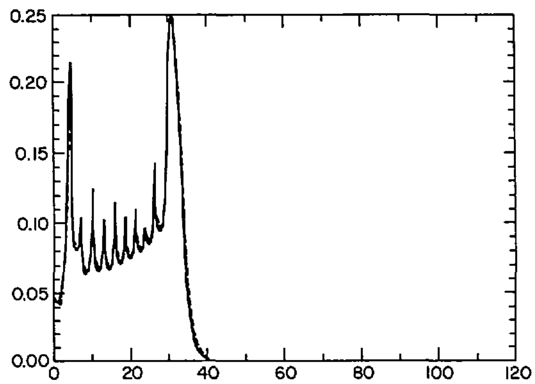

Fig. 5. Variation of conditional variance with respect to λ versus time. The dashed line curve corresponds to $\lambda=0.15$, all other parameters are the same.

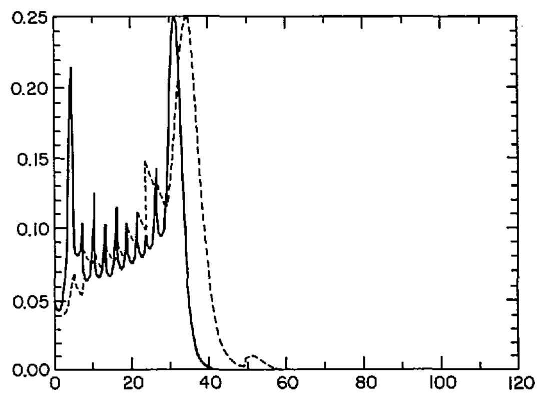

Fig. 6. Variation of conditional variance with respect to μ versus time. The dashed line curve corresponds to $\mu=1.5$, all other parameters are the same.

To analyze the filter sensitivity to parameter variations we examined the variables of the filter output (i.e., the conditional distribution of the switching time). Our computations demonstrate negligible variations with large variations in the filter parameters λ, μ, σ (we tried variations as large as 50 percent!) and almost no sensitivity to τ. Furthermore, since the quality of the estimator is judged by the conditional error variance

$$V_t=E\{(x_t-\hat{x}_t)^2|\mathcal{F}_t\}=\hat{x}_t(1-\hat{x}_t) \tag{5.1}$$

we studied also variations in V_t under similar variations in the parame-

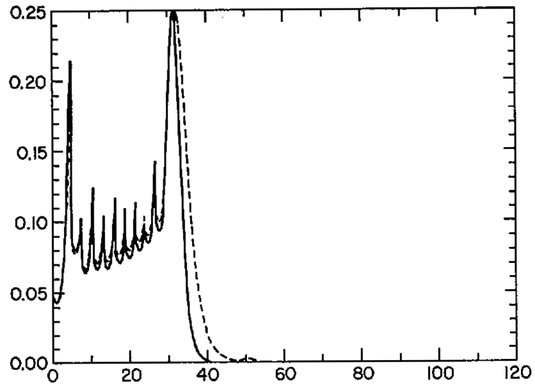

Fig. 7. Variation of conditional variance with respect to σ versus time. The dashed line curve corresponds to $\sigma^2 = 0.2521$, all other parameters are the same.

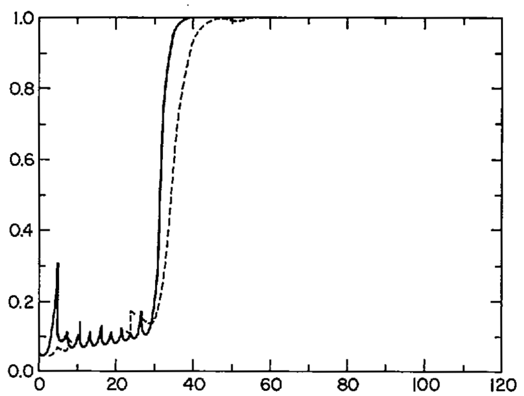

Fig. 8. Variation of conditional mean with respect to μ versus time. The dashed line curve corresponds to $\mu = 1.5$, all other parameters are the same.

ters. Again, the observed variations in V_t were minute. In particular, the result of the maximum jump estimator was almost unaffected. Representative results are shown in Figs. 5–8.

The filter appears to be most sensitive to variations in μ. Several bounds and analytical expressions of the sensitivity of V_t with respect to λ, μ, σ, can be found in [19]. In conclusion, the filter appears to be very robust, although we have not as yet obtained a complete mathematical proof of this.

VI. Conclusions

The estimator for platoon passage time developed in this paper appears to be effective, based on our simulation results. This estimator would also provide good delayed estimates of the queue at upstream traffic signals, provided the street configuration is favorable. Furthermore, the model developed for headway statistics has potential value in other traffic situations, such as incident detection.

Traffic estimates of this type are most useful if they can be used to improve traffic control. Our current research centers on the use of the models described in this and its companion paper to develop improved closed-loop traffic controls for single intersections and to coordinate groups of intersections and large networks for improved operation. In closing, we mention that similar problems appear in other types of queuing networks (such as computer or communication networks) where similar techniques can be fruitfully applied.

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Generalized Quadratic Weights for Asymptotic Regulator Properties

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***Abstract*—This paper describes a generalized quadratic weight selection procedure based on asymptotic modal properties of linear-quadratic regulators as control weights tend to zero. Explicit formulas are developed for weighting matrices which produce an asymptotic eigenstructure consisting of $p \leq n-m$ finite modes, with the rest tending to infinity in selectable Butterworth patterns or determined by other secondary design considerations.**

I. Introduction

While it is often said that the linear-quadratic regulator problem is "solved" from the point of view of theory, there still seem to be things to learn from the point of view of serious application. Probably the most important area about which better understanding is needed is the relationship between the weighting parameters selected for the basic scalar performance index and the resulting regulator properties. Practitioners have wrestled with this relationship for nearly two decades, trying various intuitive ways to select a "good set of weights" to satisfy their various design specifications. These ways range from diagonal inverse-square weighting [1], to local quadratic equivalence [2], to several versions of model following [3]–[5].

In a recent publication [6], Harvey and Stein suggested yet another scheme for approaching the weight selection problem. This new approach makes use of known relationships between the weighting parameters and the resulting asymptotic eigenstructure of the regulator as control weights tend to zero. Simple, explicit formulas were given for

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