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Dynamic, Optimal Sensor Scheduling and Value of Information

Dipankar Maity and John S. Baras
Department of Electrical and Computer Engineering, Institute for Systems Research
University of Maryland College Park, Maryland, USA 20742
Email: {dmaity, baras}@umd.edu

***Abstract*—In this paper, we present a method for the optimal state estimation of a linear system, observed by various dynamically schedulable and distributed sensors. We consider this problem to be a problem of information fusion, where the information is obtained from the sensors in exchange of scheduling and sensor operational costs. Sensors over a distributed network can work together efficiently in order to maximize the overall network performance. We consider several costs related to sensor scheduling such as the continuous cost for keeping the sensor on, and sensor switching cost for turning a sensor on from off state and vice versa. Our goal is to estimate the state as closely as possible while minimizing the scheduling cost. We incorporate the error in estimation as a cost in the optimization function. The resultant problem is then converted into an optimal control problem, where optimal scheduling is obtained by pointwise maximizing the Hamiltonian of the system.**

I. Introduction

The advancements in wireless sensor networks have made a great impact on recent technological growth in large interconnected distributed systems. Sensor networks in a distributed system consist of a family of sensors distributed over the space to monitor physical and environmental conditions. With the advancements in sensing and pervasive computing, the sensor nodes in a sensor network not only monitor the system, but they also communicate, fuse information, and oftentimes work as decision makers.

Classical estimation theory estimates the state of a system from given measurements by computing the posterior density of the process state conditioned on the available measurements. In a distributive setup the state estimate is done distributively over the network. The distributive architecture often imposes restrictions on the sensor measurements, data processing, and communication when there are only limited network resources available. The restrictions could be of the form that one sensor can sense in different modes and the sensor scheduler can choose only one such mode independently, or one can have restrictions on the communication subsystem such as only a single sensor can communicate at a given time slot, or there could be situations where it is not possible to use multiple sensors in the system such as systems using echo-based sensors like ultrasonic sensors.

Sensor scheduling emerges as a problem of selecting some of the sensors from a pool of available sensors. The optimality in this scheduling is determined by the measurements obtained from the sensors or the information retrieved from these measurements and the cost incurred in that scheduling. In most of the cases, fusion of the data emerging from multiple-sensors is done with a hope to get a better estimate than that obtained from the sensor with the least variance. In scheduling, we will have the freedom to choose the right set of sensors at any time and fuse the information in a proper way that will reduce the error in estimation.

The optimal sensor scheduling problem for state estimation and filtering has been an important research topic for a few decades. Filtering for a nonlinear diffusion process has received considerable attention [1], [2]. In the theory of nonlinear diffusion processes, the state of a system is driven by the nonlinear drift present within the system and a diffusion caused by an additional Wiener process that is present in the dynamics of the system [2]. Optimal scheduling has been studied as a problem of optimal stopping time in [3]. [4] also considered a state estimation problem with sensor scheduling; and the work considered noise and uncertainty models which are assumed to be unknown deterministic functions that satisfy energy type constraints. In [5], the authors considered the optimal sensor scheduling problem as a controlled hidden Markov model with continuous state, observation and action spaces. Their approach is simulation based where a stochastic gradient based algorithm is used to generate the optimal scheduling. Another work [6] from the same year considered a discrete time linear Gaussian process and derived the periodicity policy in the situation where the decision is whether the sensor should transmit or not. [7] also considered a linear Gaussian system and proposed a stochastic scheduling policy that is computationally tractable. In a stochastic hybrid linear Gaussian setup [8] proposed an algorithm for sensor scheduling. [9] proposed a sensor scheduling and MAP state estimate approach for hidden Markov models.

Recent works like [10] and [11] have modeled the data fusion process in a sensor network as a trust based process. These approaches estimate the state by taking convex combinations of the measurements available at the sensor nodes. The weight on each measurement is calculated based on the trust values. Though these approaches can suppress the measurements from noisy sensors by putting less weights on the measurements obtained from them, they do not perform sensor scheduling.

In this paper, we investigate the sensor scheduling problem for a continuous time linear Gaussian system in a more general setup. We will consider the switching cost in our formulation. Most of the previous methods have considered discrete time which deal with time instances, however, scheduling in continuous time requires special care to carry on the analysis.

The rest of the paper is organized as follows: Section-II,

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formulates the general problem that we aim to solve, Section-III provides basic tools to perform a change in probability measure, Section-IV provides the pointwise necessary condition for an optimal scheduling policy. The simulation results are shown in Section-V.

II. Problem Formulation

Let us consider a linear quadratic Gaussian system whose state is $x(\cdot) \in \mathbb{R}^n$ and the dynamics of this system are given in equation (1) below:

$$\begin{aligned} dx &= Axdt + QdW_t \\ x(0) &= x_0 \end{aligned} \tag{1}$$

where the matrices A, Q are in general time varying and W_t is a standard Wiener process. To maintain brevity, we will not show the explicit time dependencies of any of the system parameters in the following sections.

We have M sensors and their observations are described in equation (2)

$$\begin{aligned} dy^i &= H_i x dt + R_i dv_t^i \qquad \forall i = 1, 2, \cdots, M \\ y^i(0) &= 0 \end{aligned} \tag{2}$$

where H_i is a time varying matrix, observation $y^i \in \mathbb{R}^{d_i}$ and the noise v_t^i is a Wiener process in $\mathbb{R}^{d_i}$ which is independent of W_t. The matrices Q, R^i determine the variance of the state x and the observation y^i respectively. Note that the sensor variance of the i-th sensor will be completely characterized by the matrix R^i. We are interested in estimating some function ($\psi : \mathbb{R}^n \to \mathbb{R}$) of the state variable x at each time. For our estimation, we have M available sensors but we are free to choose any combination of them at any time. Let the estimation be denoted by $\hat{\psi}(t)$. Our goal is to jointly select the measurements to use and the estimation algorithm in such a way that the error in estimation is minimized. Now, to choose how to define the error in estimation, we consider the variance of this estimation i.e. $E(\psi(x(t)) - \hat{\psi}(t))^2$. The goal will be to minimize this variance subject to additional constraints and considerations. It is true that for any two sensors with same detectability, if one of the observations is less noisy or equivalently if the sensor has smaller variance, the estimation of that sensor will be better. The sensor measurements are the information available to us for the estimation, and the *value of the information* at time t is directly related to the reduction in the estimation error that is due to the use of specific sensors and of their measurements at time t.

In order to get the best estimate, we need to use all the sensors but that will incur more cost. Since we are interested in scheduling, it is clear that we will neither use all the sensors at any specific time, nor the measurements from a specific sensor at all times. Rather the sensors will be scheduled and their measurements will be used in such a way that the estimate $\hat{\psi}$ will be satisfactory and the cost for operating these sensors will be also minimized. In the cost of the sensors, we will also consider the switching cost i.e. sensor activation and deactivation costs. Let us denote the cost of operating sensor i at time t and state x to be $c_i(t, x)$ and the cost of switching on and off sensor i at time t and state x is $s_1^i(t, x)$ and $s_0^i(t, x)$ respectively. Let $u(t)$ be a binary vector that denotes the state of the sensors at time t. $u_i(t) = 0$ if sensor i is off and it is 1 if sensor i is on. Since there are M sensors, precisely $N = 2^M$ possible schedules can be there at any time. Let us denote $u(t)$ as a scheduling policy for our problem. The set of all possible sensor configurations at any time is given by η. For any $\nu \in \eta$, we define a set N_ν that contains the indexes of ν which have the value 1, i.e. $N_\nu = \{j | \ \nu_j = 1\}$. Let us consider that at time t the scheduling is changed from ν to ν_1 and hence the cost incurred by this action is given in equation (3).

$$k_{\nu\nu_1}(t, x) = \Sigma_{\{j \in N_\nu\}\{j \notin N_{\nu_1}\}} s_o^j(t, x) + \Sigma_{\{j \notin N_\nu\}\{j \in N_{\nu_1}\}} s_1^j(t, x) \tag{3}$$

On the other hand, the total cost for keeping the sensors on at time t with the state as x and schedule ν is as follows:

$$C_\nu(t, x) = \Sigma_{j \in N_\nu} c_j(t, x) \tag{4}$$

Our goal is to minimize the estimation error in estimation and also minimize the cost due to the sensor scheduling. Therefore, given the observations, the objective function we are set to optimize is given by equation (5) below:

$$J(u, \hat{\psi}) = E\Big[\int_{t=0}^{T} [(\psi(x(t)) - \hat{\psi}(t))^2 + C_u(t, x(t))]dt + \Sigma_j k_{u(\tau_{j-1}), u(\tau_j)}(t, x) \chi_{(\tau_j < T)}\Big] \tag{5}$$

where τ_i is the time when at least one sensor has changed its state. That is the time instances τ_i are the switching instances in the sensor schedule.

Since the sensor schedule determines the observations available at time t, we can write the available observations as a function of the sensor schedule and that is represented in equation (6).

$$dy(t, u(t)) = H(u(t))xdt + R(u(t))dv \tag{6}$$

where $H(u(t)) \in \mathbb{R}^{D \times n}$ and $R(u(t)) \in \mathbb{R}^{D \times D}$ are the matrices given in (7) and (8) respectively where $D = d_1 + \cdots d_M$, and v is the standard D dimensional Wiener process.

$$H(u(t)) = \begin{bmatrix} H_1 u_1(t) \\ \vdots \\ H_i u_i(t) \\ \vdots \\ H_M u_M(t) \end{bmatrix} \tag{7}$$

$$R(u(t)) = \begin{bmatrix} R_1 u_1(t) & \cdots & 0 & 0 \\ \vdots & & & \\ \cdots & R_i u_i(t) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & R_M u_M(t) \end{bmatrix} \tag{8}$$

If sensor i is not active at time t, the i-th row of $H(u(t))$ and $R(u(t))$ is identically zero and hence the measurement is not available in the $y(t, u(t))$ vector. Note that we are interested in performing estimation while at the same time employing sensor scheduling as a technique to minimize the cost in estimation. To keep our problem tractable in this paper we will only consider $\psi(x) = x$, i.e. we are interested in estimating the state itself. We will also consider that the sensor running costs and switching costs are only functions of time; that is, we do not consider models where these costs depend on the state x. More general models, following the general framework of [3] will be considered elsewhere.

III. Preliminaries

Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space where Ω is the set of events, $\mathcal{F}$ is the σ-algebra formed by the elements of Ω, and $\mathcal{P}$ is the probability measure. We denote the $\sigma-$algebra generated by the process $y(.,u(.))$ up to time t as $\mathcal{F}_t^{y(.,u(.))} = \sigma(y(s,u(s)), s \leq t)$. Clearly $\mathcal{F}_t^{(y(.,u(.)))}$ is an increasing sequence of σ-algebras.

For a diffusion process [12] given by (1), the infinitesimal generator of that process is given by the operator L.

$$L = \Sigma_{i,j=1}^n (QQ^T)_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \Sigma_{i=1}^n (Ax)_i \frac{\partial}{\partial x_i} \tag{9}$$

The adjoint operator of this operator is given by L^*.

$$L^*(m) = \Sigma_{i,j=1}^n \frac{\partial^2((QQ^T)_{ij} m)}{\partial x_i \partial x_j} - \Sigma_{i=1}^n \frac{\partial}{\partial x_i}((Ax)_i m) \tag{10}$$

The Fokker-Plank equation [12] for the unnormalized probability density $\mu(x,t)$ of the state x at time t, for the process (1) satisfies the partial differential equation (11):

$$\frac{\partial}{\partial t}(\mu(x,t)) = L^*(\mu(x,t)) \tag{11}$$

If $\int \mu dx = C \neq 1$, then we take the normalized density to be $\mu(x,t)/C$.

Under observations, the unnormalized conditional distribution of the process x is given by the Zakai equation [13]:

$$d\mu = L^* \mu dt + \mu x^T H(u(t))^T dy(t, u(t)) \tag{12}$$

The contents presented here will be needed to carry out the expectation that is present in the cost functional.

IV. Optimal Scheduling Policy

In this section, we will mainly develop our theory for optimal sensor scheduling and for that purpose, let us begin with considering the process

$$v(t) = \xi(t) - \int_0^t R(u(s))^{-1} H(u(s)) x(s) ds \tag{13}$$

where ξ is a standard Wiener process under the measure $\mathcal{P}$. Using Girsanov's theorem to change probability measure [14], we can say that v will be a standard Wiener process under the measure $\mathcal{P}^u$. The change of probability measure from $\mathcal{P}$ to $\mathcal{P}^u$ is given by,

$$\left.\frac{d\mathcal{P}^u}{d\mathcal{P}}\right|_{\mathcal{F}_t} = \zeta(t) \tag{14}$$

where $\zeta(t)$ is given by,

$$\zeta(t) = exp\big[\int_0^t (R(u(s))^{-1} H(u(s)) x(s))^T d\xi(s) - \frac{1}{2}\int_0^t \|R(u(s))^{-1} H(u(s)) x(s)\|^2 ds\big] \tag{15}$$

If ξ is a Wiener process that is independent of the process W_t defined in (1), W_t remains a standard Wiener process under this new measure P^u. The initial state, x_0, of the process x is independent of the process W_t and v; and as a consequence, the probability measure of x does not change under this new measure $\mathcal{P}^u$. This change of probability measure is needed to carry out the optimization since the cost functional contains terms related to the process x and the estimate $\hat{x}$ which is based on the given measurements $y(.,u(.))$. The process $y(.,u(.))$ is characterized under the probability measure $\mathcal{P}^u$.

Therefore, considering the measure $\mathcal{P}^u$ and the invariance of the probability laws of x under this measure, we can rewrite the optimization problem as follows:

$$\min_{u(\cdot),\hat{x}(.)} E^u\big[\int_{t=0}^T [(x(t)-\hat{x}(t))^2 + C_u(t)]dt + \Sigma_j k_{u(\tau_{j-1}),u(\tau_j)}(t)\chi_{(\tau_j<T)}\big] \tag{16}$$

Consequently, (16) can be written as

$$\min_{u(\cdot)}\Big[\min_{\hat{x}(\cdot)} E^u\big[\int_{t=0}^T [(x(t)-\hat{x}(t))^2 + C_u(t)]dt + \Sigma_j k_{u(\tau_{j-1}),u(\tau_j)}(t)\chi_{(\tau_j<T)}\big]\Big] \tag{17}$$

We also assume that the switching cost is always bounded from below. Under this assumption, any admissible scheduling $u(\cdot)$ can have only finite number of switchings (if the time horizon $[0,T]$ is finite) otherwise the cost for a schedule with infinite switchings will be infinite.

For any admissible schedule $u(\cdot)$, it is not hard to prove that the estimate $\hat{x}(t)$ is the estimate of the Kalman filter which minimizes the cost (19). Therefore, $\min_{\hat{x}(\cdot)} E^u(x(t)-\hat{x}(t))^2 = tr(P(t))$, where $tr(\cdot)$ is the trace of a matrix and $P(t)$ is the covariance of the process $x(t)-\hat{x}(t)$. $P(t)$ satisfies the Riccati differential equation (18).

$$\dot{P} = AP + PA^T + QQ^T - PH(u(t))^T R(u(t))^{-1} H(u(t)) P \tag{18}$$

$P(t)$ depends on the choice of scheduling u_0^t where $u_0^t = \{u(s)|\ 0 \leq s \leq t\}$ and therefore it should be denoted as $P(t, u_0^t)$; but for brevity, in what follows next, we will denote it as $P(t)$, the dependence of P on scheduling $u(\cdot)$ is implicitly assumed. Therefore, the optimization problem becomes,

$$\min_{u(\cdot)}\big[\int_{t=0}^T [tr(P(t,u_0^t)) + C_u(t)]dt + \Sigma_j k_{u(\tau_{j-1}),u(\tau_j)}(t)\chi_{(\tau_j<T)}\big] \tag{19}$$

Let us define a new function $K_\delta(t,u(t)) = \delta(t) k_{u(t^-),u(t)}(t)$. To see that this transformation exactly gives the switching cost, let $u(\tau_{j-1}) = \nu$ and $u(\tau_j) = \nu_1$ and hence using $u(\tau_j^-) = \nu$,

$$\int_s^{\tau_j^+} K_\delta(t,u(t))dt = k_{(\nu,\nu_1)}(\tau_j)$$

where $s \in (\tau_{j-1}, \tau_j]$. With this transformation, the discrete cost at switching instances can be converted to an integral cost. The cost in (19) can be rewritten as,

$$\min_{u(\cdot)} \int_{t=0}^T [tr(P(t)) + C_u(t) + K_\delta(t,u(t))]dt \tag{20}$$

Since P is a matrix that satisfies the Riccati differential equation (18), we can write the same system (18) as a scalar nonlinear system in $\mathbb{R}^{\frac{n(n+1)}{2}}$ as described by equation (21) below:

$$\dot{q} = f(t, q, u) \tag{21}$$

where $q \in \mathbb{R}^{\frac{n(n+1)}{2}}$, and the q_is are the elements of the P matrix (P being symmetric, we need only $n(n+1)/2$ elements to uniquely represent it). Let us also define $\tilde{q}$ such that

$$\dot{\tilde{q}} = L(t, q, u) \qquad \tilde{q}(0) = 0 \tag{22}$$

where $L = tr(P(t)) + C_u(t) + K_\delta(t, u(t))$. Therefore, the cost functional takes the form (23),

$$J(u(\cdot)) = \tilde{q}(T) \tag{23}$$

Thus, we have converted the Lagrange optimization problem into Mayer form [15].

Theorem IV.1. *If $u^*(.)$ is an optimal scheduling law and q^* is the corresponding optimal trajectory, then there exists a costate trajectory $p(.) : [0, T] \to \mathbb{R}^{n(n+1)/2}$ such that*

$$\dot{q}^* = \frac{\partial \mathcal{H}}{\partial p}(t, q^*, u^*, p, p_0) \tag{24}$$
$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q^*}(t, q^*, u^*, p, p_0)$$

where $\mathcal{H}(t, q, u, p, p_0) = p^T f(t, q, u) + p_o L(t, q, u)$ and $\mathcal{H}(t, q^*, u^*, p, p_0) = \max_u \mathcal{H}(t, q^*, u, p, p_0)$

Proof: To prove this, let us first define a new system $\eta = [\tilde{q}, q^T]^T \in \mathbb{R}^{1+\frac{n(n+1)}{2}}$ that satisfies the dynamics written in equation (25)

$$\dot{\eta} = \begin{bmatrix} L(t, q(t), u(t)) \\ f(t, q(t), u(t)) \end{bmatrix} = g(t, \eta(t), u(t)) \tag{25}$$

If $u(\cdot)$ is an optimal scheduling law, the trajectory of the system (21) i.e. q, can be reconstructed from the trajectory of the higher dimensional system η by simply projecting it to the proper $\frac{n(n+1)}{2}$ dimensional subspace. It should also be noted that any optimal scheduling law has to be piecewise constant otherwise the cost in (19) will be infinite.

To check the optimality of the scheduling law, we will perturb the system with slight variation in the control and analyze its behavior. Since we are interested in piecewise constant scheduling, we will introduce a small pulse in the control law and see the effects caused by this perturbation. This variation in scheduling law is given below in (26)

$$u_{w,I}(t) = \begin{cases} u^*(t), & \text{if} \quad t \notin I \\ w, & \text{if} \quad t \in I \end{cases} \tag{26}$$

where $u^*(t)$ is the optimal scheduling for the sensors, and the corresponding optimal trajectory generated is η^*. The perturbation $w \in U$ is an admissible control input and the perturbation interval I is an interval of length $a\epsilon$ with one endpoint at b, i.e. $I = [b - a\epsilon,\ b]$, where $a > 0$. Using simple Taylor series expansion, as done in [16], one can easily find the relation between the perturbed trajectory $\eta(t)$ and the optimal trajectory η^* to be,

$$\eta(b) = \eta^*(b) + \Delta g(b, w)\epsilon a \tag{27}$$

where $\Delta g(b, w) = g(b, \eta^*(b), w) - g(b, \eta^*(b), u^*(b))$. We are interested to find the effects that this perturbation has on future times i.e. $t \geq b$. Let us denote

$$\eta(t) = \eta^*(t) + \epsilon\phi(t) + o(\epsilon) \tag{28}$$

where $o(\epsilon)/\epsilon \to 0$ as $\epsilon \to 0$ and $\phi(b) = \Delta g(b, w)a$. It is not hard to show that $\phi(t)$ satisfies the differential equation (29) with initial condition $\phi(b) = \Delta g(b, w)a$ ([16], Section 4.2.4),

$$\dot{\phi} = \begin{bmatrix} 0 & L_q(t, q^*, u^*)^T \\ 0_{n\times 1} & f_q(t, q^*, u^*) \end{bmatrix} \phi \tag{29}$$

where $L_q = \frac{\partial L}{\partial q}$ and $f_q = \frac{\partial f}{\partial q}$.
The solution to the system (29) is given by $\phi(t) = \Phi(t, b)\phi(b)$; $\Phi(\cdot, \cdot)$ is the state transition matrix of the system (29). Thus, we get for any time $t \geq b$,

$$\eta(t) = \eta^*(t) + a\epsilon\Phi(t, b)\Delta g(b, w) + o(\epsilon) \tag{30}$$

Therefore the perturbation in control has caused a perturbation in the final state $\eta(T)$ by an amount $a\epsilon\Phi(T, b)\Delta g(b, w)$. It is clear that the direction of this perturbation depends only on the values of b and w, and therefore, we define a ray $\overrightarrow{\rho}(b, w)$ to denote the direction of this perturbation. We also let $\overrightarrow{P}$ denote all possible such rays for different values of b and w. This set $\overrightarrow{P}$ denotes a cone with vertex at $\eta^*(T)$ and in general this is not a convex cone. By concatenating all possible perturbations, we can generate a larger cone that contains all possible convex combinations of points in $\overrightarrow{P}$ [16]. As referred in [16], we will also name this cone to be the Terminal Cone ($co(\overrightarrow{P})$) with vertex at $\eta^*(T)$.

Let us define a system z whose dynamics are as in equation (31)

$$\dot{z} = \begin{bmatrix} 0 & 0_{n\times 1} \\ -L_q(t, q^*, u^*) & -f_q(t, q^*, u^*)^T \end{bmatrix} z. \tag{31}$$

Clearly, the variables z and ϕ are adjoint to each other and hence, we have

$$(\dot{z^T\phi}) = z^T\dot{\phi} + \dot{z}^T\phi = 0 \tag{32}$$

As a consequence of this adjoint property, $z^T(t_1)\phi(t_1) = z^T(t_2)\phi(t_2)$ for all $t_1, t_2 \geq b$ (since ϕ is defined for $t \geq b$).

Now we define a ray $r \in \mathbb{R}^{1+\frac{n(n+1)}{2}}$ in the direction $[-1, 0, \cdots, 0]$ starting at $\eta^*(T)$, and clearly this ray does not intersect with the convex cone $co(\overrightarrow{P})$. To see this let us consider the optimality of η^*, which says $r^T(\eta(T) - \eta^*(T)) = \tilde{q}^*(T) - \tilde{q}(T) \leq 0$.
By the separating hyperplane theorem [17], we know that two convex sets can be separated by a hyperplane i.e. for any two convex sets, there exists a hyperplane such that the two convex sets lie in two different sides of the separating hyperplane. Note that the convex set $co(\overrightarrow{P})$ and ray r share a common point namely $\eta^*(T)$ and hence the separating hyperplane should pass through this point. If the normal to the separating hyperplane is $\begin{pmatrix} p_0^* \\ p^*(T) \end{pmatrix}$, then we have that

$$\begin{pmatrix} p_0^* \\ p^*(T) \end{pmatrix}^T r \geq 0 \tag{33}$$

and

$$\begin{pmatrix} p_0^* \\ p^*(T) \end{pmatrix}^T (\eta(T) - \eta^*(T)) \leq 0 \tag{34}$$

From (33), we get $p_0^* \leq 0$. Let us denote the elements of the vector z defined in (31) be such that the first component of z is p_o and the rest $n(n+1)/2$ components be denoted by p i.e. $z(t) = \begin{pmatrix} p_0^* \\ p^*(t) \end{pmatrix}$. The system (31) is a linear system and we have not specified any initial or final condition for this system. At this point we specify that $z(T) = \begin{pmatrix} p_0^* \\ p^*(T) \end{pmatrix}$

Since $\begin{pmatrix} p_0^* \\ p^*(t) \end{pmatrix} = z(t)$ and ϕ are adjoint to each other, therefore, $\begin{pmatrix} p_0^* \\ p^*(T) \end{pmatrix}^T \phi(T) = \begin{pmatrix} p_0^* \\ p^*(t) \end{pmatrix} \phi(t)$

Therefore, from (34) and (28),

$$\begin{pmatrix} p_0^* \\ p^*(T) \end{pmatrix}^T \phi(T) = \begin{pmatrix} p_0^* \\ p^*(b) \end{pmatrix}^T \phi(b) \leq 0 \tag{35}$$

Substituting the expression of $\phi(b)$ in (35), we obtain

$$\begin{pmatrix} p_0^* \\ p^*(b) \end{pmatrix}^T (g(b, \eta^*(b), w) - g(b, \eta^*(b), u^*(b))) \leq 0 \tag{36}$$

Defining the Hamiltonian $\mathcal{H}$ to be $\mathcal{H}(q, u, p, p_0) = pf(t, q, u) + p_0 L(t, q, u)$ we obtain from (36),

$$\mathcal{H}(q^*(b), u^*(b), p^*(b), p_0^*) \geq \mathcal{H}(q^*(b), w, p^*(b), p_0^*) \tag{37}$$

■

Equation (37) is the well known Pontryagin maximum principle equation [18]. Therefore, the functional optimization problem (19) has been converted into a pointwise optimization problem, which can be solved along with the constraints on the scheduling law $u(\cdot)$.

V. Simulation Results

Let us consider the linear stochastic system given by the dynamics:

$$dx = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} xdt + \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{bmatrix} dW_t \tag{38}$$

For this system, we have three observations y_1, y_2, and y_3 whose dynamics are given below:

$$dy_1 = [1,\ 0]xdt + \sqrt{.2}dv_t^1$$

$$dy_2 = [0,\ 1]xdt + \sqrt{.2}dv_t^2$$

and

$$dy_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} xdt + \begin{bmatrix} .5 & 0 \\ 0 & \sqrt{.1} \end{bmatrix} dv_t^3 \tag{39}$$

y_3 is a full state observation where y_1 and y_2 partially observe the states. The running costs for the sensors are taken to be constant and their values are $c_1 = 0.05$, $c_2 = 0.07$ and $c_3 = 0.1$. The switching cost is also taken to be time independent and its value is $k = 0.05$. Moreover, we also restrict that the switching cost is zero when a sensor is going from on state to off state. Note that the third observation gives full state information but it has high uncertainty. The sensor costs are taken in such a way that the sensor with low variance and high observability has higher cost. For this system, the optimal schedule for the sensors is shown in Figure 1.

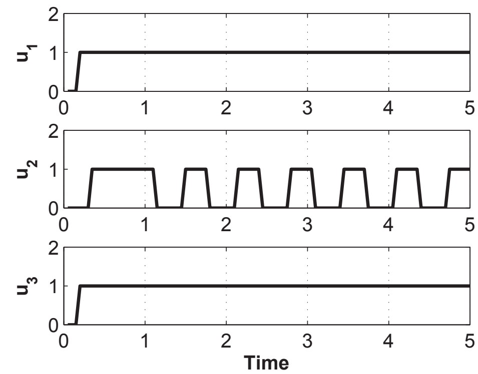

Fig. 1. Optimal schedule for the Sensors.

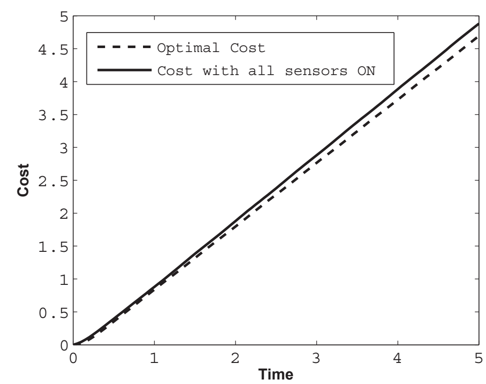

Fig. 2. Optimal cost versus cost of operating three sensors simultaneously.

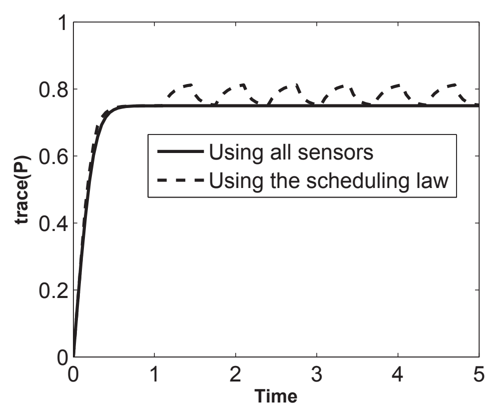

Fig. 3. $tr(P)$ for optimal scheduling and for the case when all sensors are active.

The figures show that the optimal scheduling does not require all the sensors to be operating all the time. With this scheduling scheme, the optimal cost and the cost incurred when all sensors are functioning are compared in figure 2. The covariance matrix P is an indication for the accuracy in estimation. In figure 3, the $tr(P)$ is plotted for the optimal scheduling and the case when all sensors are operating.

VI. CONCLUSION

In this paper, we have considered a distributed dynamic sensor scheduling problem for the purpose of state estimation. The error in state estimation is denoted by the trace of the covariance matrix between the process x and the estimate $\hat{x}$. The σ-fields generated by the measurements are dependent on the choice of scheduling and hence proper care has been taken by preforming change of probability measure to evaluate the error covariance. Though it is not shown in this paper, direct calculation leads to the fact that the covariance matrix satisfies the Riccati differential equation given in equation (18). The cost functional after minimizing w.r.t. the estimate $\hat{x}$, becomes a standard Lagrangian cost with the new definition of the switching cost defined in equation (3). Necessary conditions for optimality are derived and the functional optimization problem (19) has been converted to a pointwise optimization problem (Theorem IV.1).

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